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Electron–Muon Mass Ratio and the Masses of Their Neutrinos

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In this note, we arrange equal mass for all the four leptons, e, μ, and their neutrinos through their coupling to a Higg's quartet. In addition, the electron and muon are coupled to the left handed and right handed Higgs doublets. This is a pseudo scalar coupling. This enables these charged leptons to attain different masses. Their masses are arranged to be proportional to their neutrino mass. The mass of the electron or muon neutrino turns out to be 6.3 eV.

Recently we derived theoretical expressions for the masses of all known leptons and quarks (Raju, 1987). These expressions were deduced on dimensional grounds. In particular we found that,

$$m_e^2 = mM_{WL} \frac{(g_V/g_A)^4_{\nu_e}}{(g_V/g_A)^4_{e\mu}} \left[1 - \left(1 - \left(\frac{g_V}{g_A} \right)^4_{eu} \right)^{1/2} \right] \tag{1}$$

and

$$m_\mu^2 = mM_{WL} \frac{(g_V/g_A)^4_{\nu_e}}{(g_V/g_A)^4_{e\mu}} \left[1 + \left(1 - \left(\frac{g_V}{g_A} \right)^4_{e\mu} \right)^{1/2} \right] \tag{2}$$

Here $(g_V)_{e\mu}$ and $(g_A)_{e\mu}$ are vector and axial vector couplings of the neutral Z boson with the electron or muon. The above expressions yield excellent ratio of m_e^2 and m_μ^2 if we consider standard model prescriptions for $(g_V)_{e\mu}$ and $(g_A)_{e\mu}$ with

$$x_L = \frac{e^2}{g_L^2} = x_W = 0.2254$$

Here x_L (or x_W) is the Weinberg mixing parameter.

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Suppose the electroweak model is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. In that case, we will have two neutral currents and in addition to the standard Z, we will have one more neutral boson known as D-boson. We have also developed this gauge model with two neutral currents in such a way that the interaction contains two neutral currents and is free of triangle anomalies (Raju, 1985). In Raju (1987) we have shown that we obtain the same numbers for m_e^2 and m_μ^2 even if we use the $(g_V)_{e\mu,D}$ and $(g_A)_{e\mu,D}$ in place of $(g_V)_{e\mu,Z}$ and $(g_A)_{e\mu,Z}$ provided $x_R=0.2746$, and $x_L=x_W=0.2254$. In all these computations we assumed that $(g_V/g_A)^2_{\nu_e}=(g_V/g_A)^2_{\nu_\mu}=1$, which in other words means that the neutrinos are strictly left-handed. The purpose of the present note is to derive equations (1) and (2) by yet another interesting method.

We assume here that ν_e and ν_μ are mass eigen states of the electron and muon neutrinos. Let $\phi(\frac{1}{2},\frac{1}{2},0)$ be a Higgs quartet, with,

$$\phi=\begin{pmatrix} u_3 & 0 \\ 0 & v_3 \end{pmatrix} \tag{3}$$

This ϕ and $\tilde{\phi}=\tau_2\phi\tau_2$ couple with ν_e and e in the following fashion.

$$\begin{aligned} L_0 &= h_3(\bar{\nu}_e,\bar{e})\frac{(1-\tau_3)}{2}\tilde{\phi}\begin{pmatrix}\nu_e\\ e\end{pmatrix} \\ &- h_3(\bar{\nu}_e,\bar{e})\frac{(1+\tau_3)}{2}\phi\begin{pmatrix}\nu_e\\ e\end{pmatrix} \\ &+ h_3'(\bar{\nu}_e,\bar{e})\frac{(\tau_1+i\tau_2)}{2}\phi\begin{pmatrix}\nu_e\\ e\end{pmatrix} \\ &+ h_3'(\bar{\nu}_e,\bar{e})\phi\frac{(\tau_1-i\tau_2)}{2}\begin{pmatrix}\nu_e\\ e\end{pmatrix} \end{aligned} \tag{4}$$

Where, we have assumed that, u_3, v_3, and h_3, h_3' to be real. There is nothing wrong to assume these to be real in so far as the mass matrix is concerned. The above Lagrangian can be summarized with the following mass matrix,

$$L_0=(\bar{\nu}_e,\bar{e})M\begin{pmatrix}\nu_e\\ e\end{pmatrix} \tag{5}$$

Where

$$M=\begin{pmatrix} -h_3u_2 & h_3'v_3 \\ h_3'v_3 & h_3u_2 \end{pmatrix} \tag{6}$$

The matrix MM^+ is automatically diagonal. Untill now ν_e and e which are mass eigen states, have equal mass m, where,

$$m=(h_3^2u_3^2+h_3'^2v_3^2)^{1/2} \tag{7}$$

There is no loss of generality if we do the same thing with ν_μ and μ where ν_μ is a mass eigen state of the muon neutrino. So by adjoining the muon contribution to L_0 the mass part now reads,

$$L_0 = (\bar{\nu}_e, \bar{e}) M \begin{pmatrix} \nu_e \\ e \end{pmatrix} + (\bar{\nu}_\mu, \bar{\mu}) M \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \tag{8}$$

Where the matrix M is same in both the expressions and is given by equation (6). The simple meaning of (8) is that, at this stage, e, μ, and their neutrinos all have the same mass m given by equation (7).

Suppose in addition to (8) e, and μ are coupled to ϕ_L and ϕ_R, where ϕ_L and ϕ_R are the Higgs doublets corresponding to the group $SU(2)_L \times SU(2)_R \times U(1)$, with

$$\phi = \phi'_L + V_L \quad \text{and} \quad \phi_R = \phi'_R + V_R$$

and

$$\langle \phi'_L \rangle = \langle \phi'_R \rangle = 0 \tag{9}$$

and that the neutrinos have no coupling whatsoever with these doublets. In other words, e and μ only coupled to ϕ_L and ϕ_R in the following fashion:

$$L'_0 = ia_L \bar{e}\gamma_5 e\phi_L + ia_R \bar{e}\gamma_5 e\phi_R + ib_L \bar{\mu}\gamma_5 \mu\phi_L + ib_R \bar{\mu}\gamma_5 \mu\phi_R \tag{10}$$

We add L'_0 and L_0 and first examine what happens to the electron mass. We have for the electron mass part,

$$m\bar{e}e + ia_L \bar{e}\gamma_5 e\phi_L + ia_R \bar{e}\gamma_5 e\phi_R \tag{11}$$

Let,

$$e = \exp(-\tfrac{1}{2} i\gamma_5 \alpha_1) e' \tag{12}$$

By using (9) and (12) in (11) we easily note that equation (11) is

$$\begin{aligned} &\bar{e}'e'[m \cos \alpha_1 + \sin \alpha_1 (a_L V_L + a_R V_R)] \\ &\quad + \bar{e}\gamma_5 e'[-im \sin \alpha_1 + ia_L V_L \cos \alpha_1 + ia_R V_R \cos \alpha_1] \\ &\quad + a_L \bar{e}'[\sin \alpha_1 + i\gamma_5 \cos \alpha_1] e' \phi'_L \\ &\quad + a_R \bar{e}'[\sin \alpha_1 + i\gamma_5 \cos \alpha_1] e' \phi_R \end{aligned} \tag{13}$$

We now define $\tan \alpha_1$ in a way such that the co-efficient of $\bar{e}'\gamma_5 e'$ is zero. In addition, there is no loss of generality, if we write,

$$a_L v_L + a_R V_R = a_0 V_L \tag{14}$$

Where,

$$a_0 = a_L \left(1 + \frac{a_R V_R}{a_L V_L}\right) \tag{15}$$

In view of (15),

$$\tan \alpha_1 = \frac{a_0 V_L}{m} \tag{16}$$

Introducing equations (15) and (16) and with a little algebra we readily note that Equation (13) is,

$$\begin{aligned} &\bar{e}' e' (m^2 + a_0^2 V_L^2)^{1/2} + a_L \bar{e}' (\sin \alpha_1 + i\gamma_5 \cos \alpha_1) e' \phi'_L \\ &+ a_R \bar{e}' (\sin \alpha_1 + i\gamma_5 \cos \alpha_1) e' \phi'_R \end{aligned} \tag{17}$$

Here a_L and a_R are real coupling constants as b_L and b_R are. The CP violation is now caused by the exchange of ϕ'_L and ϕ'_R. Equation (17) shows that the mass of the neutrino remains unchanged because it has no coupling to ϕ_L and ϕ_R, whereas the mass of the electron is given by,

$$m_e^2 = m^2 + a_0^2 V_L^2 \tag{18}$$

A similar analysis yields for the muon part,

$$\begin{aligned} &\bar{\mu} \mu' (m^2 + b_0^2 V_L^2)^{1/2} + b_L \bar{\mu}' (\sin \alpha_2 + i\gamma_5 \cos \alpha_2) \mu' \phi'_L \\ &+ b_R \bar{\mu} (\sin \alpha_2 + i\gamma_5 \cos \alpha_2) \mu' \phi'_R \end{aligned} \tag{19}$$

Where,

$$b_0 = b_L \left(1 + \frac{b_R V_R}{b_L V_L}\right) \tag{20}$$

and,

$$\tan \alpha_2 = \frac{b_0 V_L}{m} \tag{21}$$

The mass of the muon is given by

$$m_\mu^2 = m^2 + b_0^2 V_L^2 \tag{22}$$

From equations (18) and (22) it is clear that $a_0 \neq b_0$ since $m_e \neq m_\mu$. We wish to rearrange such that $a_0^2 V_L^2$ and $b_0^2 V_L^2$ contribute a term, m^2 in equations (18) and (22). With this in mind, and without any loss of generality we can write,

$$a_0^2 = \frac{m M_{WL}}{V_L^2} \left[B(1 - A) - \frac{m}{M_{WL}} \right] \tag{23}$$

and

$$b_0^2 = \frac{m M_{WL}}{V_L^2} \left[B(1 + A) - \frac{m}{M_{WL}} \right] \tag{24}$$

Where M_{WL} is the mass of the W_L boson of the standard model and B and A are still unknown constants. Inserting equations (23) and (24) in (18) and (22) we readily observe that,

$$m_e^2 = mM_{WL}B(1-A) \tag{25}$$

and,

$$m_\mu^2 = mM_{WL}B(1+A) \tag{26}$$

There is an empirical relation,

$$\frac{2m_e m_\mu}{m_e^2 + m_\mu^2} = (g_V/g_A)^2_{e\mu,Z} \tag{27}$$

where, $(g_V/g_A)_{e\mu,Z}$ is the ratio of vector to axial vector couplings of standard Z with e, μ leptons. The left-hand side of equation (27) is known because m_e and m_μ are known precisely. The right-hand side of (27) is also known since $x_W \approx 0.225$ or 0.23. This shows that (27) is an excellent fit to the experimental situation. Equation (27) can be reproduced if we choose,

$$A = \left[1 - \left(\frac{g_V}{g_A}\right)^4_{e\mu}\right]^{1/2} \tag{28}$$

This expression for A is independent of B, m or M_{WL}. Inserting the approximate value $A \approx (1 - \frac{1}{2}(g_V/g_A)^4_{e\mu})$ into (25) and after a little rearrangement we note that,

$$m \approx \frac{2m_e^2}{M_{WL}} \approx 6.3 \text{ eV}$$

if

$$B = \frac{(g_V/g_A)^4_{\nu_e}}{(g_V/g_A)^4_{e\mu}} \tag{29}$$

In the expression for B, we have taken advantage of the fact that $(g_V/g_A)^4_{\nu_e} = 1$. This is done to enable us to generalize these results for the other leptsons and quarks as well (See Raju, 1987). We have extended these results for the left-right model as well. By inserting equations (28) and (29) into (25) and (26) we obtain the expressions for m_e^2 and m_μ^2 given by equations (1) and (2).

In place of equations (15) and (20), we could have factorized $a_R V_R$ and $b_R V_R$ to obtain

$$a_0' = a_R\left(1 + \frac{a_L V_L}{a_R V_R}\right) \tag{30}$$

$$b_0' = b_R\left(1 + \frac{b_L V_L}{b_R V_R}\right) \tag{31}$$

By using, a_0' and b_0' and by defining them in a manner similar to equations (23) and (24), and (16) & (21), we can readily obtain that,

$$m_e^2 = mM_{WR}[B'(1-A')] \tag{32}$$

and,

$$m_\mu^2 = mM_{WR}[B'(1+A')] \tag{33}$$

Here the constants B' and A' cannot be identical to B and A of equations (23) and (24). But equations (34) and (35) enable us to make $m \to 0$, as $M_{WR} \to \infty$. On the other hand, if experimentally m and M_{WR} are known B' and A' can be computed from m_e and m_μ. As a special case if $A = A'$, then we have,

$$BM_{WL} = B'M_{WR} \tag{34}$$

and nothing more can be said about it.

Before concluding this note we wish to make a few comments on the neutrino. In place of ν_e and ν_μ we could have used ν_1 and ν_2 and couple them to ϕ_3 to obtain the matrix M given by (6). This matrix is a Majorana type mass matrix. It is well-known that a Dirac neutrino consists of two Majorana neutrinoes with equal masses and opposite CP properties. If we diagonalize M, instead of MM^+, we observe that,

$$\nu_e = \nu_1 \cos\psi + \nu_2 \sin\psi$$

and,

$$\nu_\mu = -\nu_1 \sin\psi + \nu_2 \cos\psi \tag{35}$$

Where,

$$\tan\psi = \frac{2h_3'v_3}{2h_3u_3} = \frac{h_3'v_3}{h_3u_3}$$

If

$$h_3'v_3 \ll h_3u_3$$

then

$$\nu_e \simeq \nu_1 \quad \text{and} \quad \nu_\mu \simeq \nu_2$$

But in our opinion it is dangerous to diagonalize M instead of MM^+. The relationship of ν_{eL} to ν_e and that of $\nu_{\mu L}$ to ν_μ is still obscure. Suppose a Majorana mass term of the sort $me^{i\theta}\nu_L^T C^{-1}\nu_L$ exists, it can be expressed in the form of a Dirac mass term i.e., $m\bar{\nu}_R\nu_L$ if $\bar{\nu}_R = e^{i\theta}\nu_L^T C^{-1}$ where C is the charge conjugation matrix. We simply do not have any proof and leave this problem of neutrinos to posterity.

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